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Computational Approaches for Recognising and Reconstructing Ancient Games: The Case of Ludus Latrunculorum

1. INTRODUCTION

THE DIGITAL LUDEME PROJECT

1.1 Approaches to Game Reconstruction

The existence of board games in the ancient world has been documented from the early days of archaeology, and along with the discovery of boards and their identification with games known from written texts, archaeologists and others interested in games have sought to produce playable rulesets to bring these ancient practices back to life. Some authors who have attempted to reconstruct the rules for ancient games have paid very close attention to the ancient sources,¹ while others have been more creative in their interpretations.² Usually, there is a degree of playtesting involved, to produce a game that seems to play well. Nevertheless, these reconstructions are subjective to varying degrees, and it is not always clear whether the rules that have been included in a particular reconstructed game are inspired from games that may be reasonably connected to their

ancient counterparts or chosen for other reasons. This paper uses the example of the ancient Roman game Ludus Latrunculorum to demonstrate the ways that a computational approach can contribute to the processes of identifying games in the archaeological record and reconstructing playable rulesets.³

1.2 The Digital Ludeme Project

The Digital Ludeme Project is a five-year research project that seeks to improve our understanding of the development of (primarily) board games throughout recorded human history, through computational analyses of the available evidence. This task is made challenging by the paucity and incomplete nature of evidence for ancient games and the unreliable nature of much of the information that is available about them, which is often based on interpretations of the source material filtered through a modern (Western) lens. Another factor is that the rules for games are typically carefully crafted and fragile to change, and there is no guarantee that even well researched reconstructions of ancient rules that are entirely plausible within their historical and cultural contexts

1 e.g., SCHÄDLER (2001); SCHÄDLER (1994).

2 e.g., BECQ DE FOUQUIÈRES (1869): 454–456; BELL (1979): 86–87.

3 BROWNE *et al.* (2019).

will actually play well *as games*.

The Digital Ludeme Project promotes a new field of research called *Digital Archaeoludology*⁴ in which games are digitally modelled as accurately as possible from the available evidence, then missing information about them is interpolated probabilistically based on how well the candidate rulesets function as games in addition to their authenticity within the given historical and cultural contexts.

This chapter demonstrates these principles in action. Using the historical and cultural context to identify plausible rule combinations for the reconstruction of Ludus Latrunculorum, we then apply computational self-play analyses of the resulting candidate rulesets to identify those that work more successfully as games. While this approach may not necessarily reveal how the game was actually played, it can indicate with some confidence how it was *not* played, which can be just as useful to know.

1.3 Ludii

Ludii is a computer program that was developed as part of the Digital Ludeme Project that constitutes a complete general game system for modelling, playing, evaluating and reconstructing the full range of games required for the project. Games are modelled digitally as structured sets of *ludemes*⁵ to provide a playable database of both complete games and partial descriptions requiring reconstruction, suitable for computer self-play by *artificial intelligence* (AI) agents which can playtest candidate rulesets much more quickly than human players.

The Digital Ludeme Project games database⁶ compiles what is known about traditional games from the past five thousand years, including geographic, chronological, social, and rules data for 1,006 games at the time of writing.⁷ Compiling all of this data facilitates comparison between games, particularly in connecting games which are close to each other geographically, chronologically, and with respect to game concepts. In the future, the process of connecting games according to these measurements will be automated, but for this study with a limited scope an *ad hoc* approach must be taken and is sufficient to make the necessary connections.

1.4 Game Concepts

Game concepts⁸ are features designating high-level aspects of games which can be shared between different games. Each concept is expressed in game-terms commonly used by players and designers.

They can be related to the equipment (board or pieces), or rules (game setup, ending condition(s), or movement of the pieces), but also to more general aspects such as properties (e.g., time model, information type, symmetries). The current list of all identified concepts used through Ludii is available on our website.⁹

Thanks to the ludeme representation of games in Ludii, during the compilation process, the existence of specific ludemes or combinations of ludemes trigger each of these concepts. Binary concepts are activated by the existence of ludemes while numerical concepts instead have their values set when the game is compiled. Consequently, all concepts are computed in a few milliseconds. For this reason, they are a powerful tool to identify similar games.

2. LUDUS LATRUNCULORUM: THE EVIDENCE

2.1 Documentary Evidence for Rules

Ludus Latrunculorum, or the “Game of Little Soldiers”, is one of the games played by the Romans about which we have the most documentary evidence, since several authors seem to have been familiar enough with the game to mention portions of the rules. A complete recounting of the textual references to the game is beyond the scope of this paper, and has been covered extensively elsewhere.¹⁰ Nevertheless, there are four aspects of the game that can be gleaned from the sources.

Placement phase: Two ancient sources confirm that the board is empty at the beginning of the game. In *Laus Pisonis*, by an anonymous author, there is a passage which praises Piso in the context of a game that holds the same military symbolism as other descriptions of the game.¹¹ It is explicitly stated that the pieces are cunningly placed on an open board.¹² Isidore, writing in the late 6th or early 7th century CE, indicates three kinds of pieces or, more precisely, the states of movement of three kinds of pieces: *ordine*, which move regularly, *vagi*, which move anywhere, and *inciti*, which cannot move.¹³

Orthogonal movement: Ovid, in his *Tristia*, indicated that the pieces on the board are moved in a straight line.¹⁴

Custodial capture: The next phrase in Ovid’s *Tristia* states that a piece between two enemies is lost.¹⁵ Ovid also makes this clear in *Ars Amatoria*, where he says

4 BROWNE *et al.* (2019).

5 A *ludeme* is a game-related concept that can be expressed as a functional unit. See PARLETT (2016).

6 Accessible at <https://ludii.games/library>.

7 STEPHENSON, CRIST, BROWNE (2020).

8 PIETTE *et al.* (2021).

9 Accessible at <https://ludii.games/searchConcepts.php>.

10 SCHÄDLER (1994).

11 Anonymous, *Panegyric on Piso*, 192–193.

12 Austin (1934), 30; SCHÄDLER (1994), 55–54.

13 Isidore, *Origins*, 18.67; SCHÄDLER (1995), 82.

14 Ovid, *Sorrows*, 2.477; SCHÄDLER (1994): 52.

15 Ovid, *Sorrows*, 2.477; SCHÄDLER (1994): 52.

that one counter perishes between twin foes.¹⁶ This form of capture is commonly known as custodial capture in games literature.¹⁷

Quadrangular board: Varro, writing about the Latin language in the 1st century BCE, describes a declination table for the word *albus*, with horizontal and vertical lines, as similar to the game board used to play *Ludus Latrunculorum*.¹⁸ This indicates that the game was played on a rectangular or square board (i.e., Types 29 and 30—see Appendix to this volume), but does not specify its size. Schädler points out that a declination table would have six lines for the six cases of Latin nouns, and six columns for the singular and plural for masculine, feminine, and neuter genders.¹⁹

2.2 Archaeological Evidence for Boards

Despite Varro's implication, only one six-by-six board has been found at a Roman site. Therefore, we must assume that Varro's description refers generally to a square or rectangular board, rather than one of a specific size. Archaeological evidence, then, must be consulted to provide information about the size of the quadrangular grid that likely formed the board of *Ludus Latrunculorum*.

Quadrangular boards within the *limes* of the empire have been found in many different kinds of archaeological contexts (Figure 1). A considerably large number have been found at fortresses and military installations, particularly in the provinces of Britannia,²⁰ Hispania Terraconensis,²¹ and Aegyptus,²² as well as in urban contexts, such as within Rome itself, Athens,²³ and the cities of Asia Minor,²⁴ in spaces devoted to leisure such as the Antonine Baths at Carthage,²⁵ as well as rural sites such as Gebel el-Silsila in Egypt²⁶ and Thornham, England.²⁷

The boards that have been preserved are made of stone or ceramic. *Ludus Latrunculorum* tends to appear as secondary use on stone surfaces, whether as graffiti or as a reappropriated object on which the board is carved. Only one example, formerly held



Fig. 1. Four Roman *Ludus Latrunculorum* boards held at Dover Castle. Clockwise from top left: Intact 8 x 8 chalk board (14.5 x 26 x 10 cm), from Dover, 2nd c. CE?, Dover Castle DV2000; fragmentary marble board with 8 or 9 x 7+ squares (26 x 28 x 4.5 cm), from Richborough Fort, Roman in date, Dover Castle 78305715; fragmentary marble board with 10+ x 5+ squares (19.2 x 23 x 4.2 cm), from Richborough Fort, Roman in date, Dover Castle 78305708/478; fragmentary sandstone board with 6+ x 5+ squares (14.5 x 10.3 x 2.6–3.3 cm), from Richborough Fort, after 280 CE, Dover Castle 78303093. Photos by Summer Courts.

in Zurich,²⁸ appears to have been manufactured as a board for *Ludus Latrunculorum*. It is questionable whether all of the grids on ceramic tiles were meant to have functioned as game boards; it has also been noted that ceramic tiles may have been scored in the same manner to aid in fixing them to vertical surfaces.²⁹ Wooden boards may also have been used to play *Ludus Latrunculorum*, but they have not survived in the archaeological record.

In addition to artifacts used for playing, there are two terracotta models of quadrangular boards which appear to represent *Ludus Latrunculorum*. One comes from Athens, with a group of players sitting around a six-by-seven board.³⁰ Another, found in Egypt's Fayyum Oasis, shows a six-by-seven board with pieces arrayed on it.³¹

Judging from the boards with an intact gaming pattern, it is apparent that there is a wide range of sizes for quadrangular boards from Roman sites. The twenty-five known intact boards (see Figure 2) show that they range in size from six-by-six to eleven-by-

16 Ovid, *The Art of Love*, 358.

17 Bell (1979), 78; Parlett (1999), 199; also called interception capture, Murray (1951), 10.

18 Varro, *On the Latin Language*, 10.22.

19 Schädler (2007), 361.

20 Austin (1934), 26–27; Schädler (1994), 50; Pace 2015; Pace 2020; Pace 2022; Courts, Penn (2019).

21 Carretero Vaquero (1998); Fernández Pintos (2017), 234–240.

22 Mulvin, Sidebotham (2004), 611–612.

23 Schädler (1994), 49–50.

24 Bell (2007), 98; Ersoy, Erdin (2015), 151.

25 De Voogt (2019), 91.

26 De Voogt, Nilsson, Ward (2020), 127–128.

27 Gregory, Gurney (1986), 13.

28 May (1991), 174–175.

29 Courts, Penn (2019): 6–7.

30 Michaelis (1863); Schädler (1994), 53.

31 Petrie (1927), 55.

Location	Size	Reference
Aventicum	6x6	Roman Museum of Avenches 13/15890-01
Athens (model)	6x7	SCHÄDLER (1994): 53
Monte de Santa Tegra	6x7	FERNÁNDEZ PINTOS (2017): 235-236
Aventicum,	6x8	Roman Museum of Avenches X/02924
Qasr Ibrim	7x8	ROSE (1996): 160
Chesters Fort	7x8	AUSTIN (1934): 27
Villa Adriana	7x8	MANDERSCHIED et al. (2011): 514-518
Corbridge	7x8	MURRAY (1913): 30
Gebel el-Silsila	7x8	DE VOOGT et al. (2020): 127
Parthenon, Athens	8x8	KARAKITSOU (2009): 24
Chedworth	8x8	BADDESLEY (1925)
Exeter	8x8	HOLBROOK and BIDWELL (1991): 278
Miletus	8x8	BELL (2007): 98
Smyrna	8x8	ERSOY and ERDIN (2015)
Smyrna	8x8	ERSOY and ERDIN (2015)
Brescia	8x8	MOSCA and PUPPO (2012): 274–275, 278
Basilica Iulia, Rome	8x8	SCHÄDLER (1994): 49-50
Xanthus	8x8	BELL (2007): 98
Kom Ombo	8x9	CRIST et al. (2016): 140-141
Gebel el-Silsila	8x9	DE VOOGT et al. (2020): 127
Carthage	9x10	DE VOOGT (2019): 91
Dover	10x10	PHILIP (1981): 167
Samos	11x12	SCHÄDLER (2007): 361
Puig Castellar de Bosca	11x16	RODRIGO REQUENA and ROMANÍ SALA (2021)

Fig. 2. Intact quadrangular boards found within the borders of the Roman Empire.

Location	Preserved Grid	Reference
Singidunum	2x2	JANKOVIC (2009): Fig. 11
Cemenelum	2x5	MOSCA and PUPPO (2012): 274–275
Saldum	3x3	JEREMIC (2009): 50–51
Didymoi	3x3	BRUN (2011): 126, 153
Cumidava	3x4	GUDEA and POP (1971): Pl. LII.3
Singidunum	3x4	JANKOVIC (2009): Fig. 12
Didymoi	3x4	BRUN (2011): 121, 143
Petavonium	4x1	CARRETERO VAQUERO (1998): Fig. 2.6
Petavonium	4x2	CARRETERO VAQUERO (1998): Fig. 2.4
Conimbriga	4x4	DA PONTE (1986): 138-139
Halpići	4x4	BUSULADŽIĆ (2017): Pl. 3
Petavonium	4x4	CARRETERO VAQUERO (1998): Fig. 2.3
Abu Sha'ar	4x5	MULVIN and SIDEBOTHAM (2004): Fig. 9.3
Abu Sha'ar	4x7	MULVIN and SIDEBOTHAM (2004): Fig. 9.2
Didymoi	4x7	BRUN (2011): 126, 153
Petavonium	5x3	CARRETERO VAQUERO (1998): Fig. 2.5
Vindolanda	5x5	PENN and COURTS (n.d.)
Krokodilo	5x7	MATELLY (2003): 594, 605
Abu Sha'ar	5x9	MULVIN and SIDEBOTHAM (2004): Fig. 9.1
Exeter	6-7x8-9	HOLBROOK and BIDWELL (1991): Fig. 135
Abu Sha'ar	6-8x8-9	MULVIN and SIDEBOTHAM (2004): Fig. 9.5
Bearsden	6x4	BREEZE (2016): 93–94
Aventicum	6x6	DANIAUX (2019): Fig. 3
Richborough	6x7	BUSHE-FOX (1928): Fig. 1.3
Aquis Querquennis	6x7	AVELAIRA (2015): 66
Noville	6x7	PÉREZ LOSADA (1993): 1058, Fig. 3
Buciumi	7x3	GUDEA (1971): Pl. LVI.4
Vindolanda	7x9	PENN and COURTS (n.d.)
Carthage	8x10-11	DE VOOGT 2019: 91
Richborough	8x5	BUSHE-FOX (1928): Fig. 1.2
San Chuis	8x5	VILLA VALDÉS (2010–2012): 109–110
Abu Sha'ar	8x6	MULVIN and SIDEBOTHAM (2004): Fig. 9.4
Petavonium	8x7	CARRETERO VAQUERO (1998): Fig. 2.1
Abu Sha'ar	9–10x10–11	MULVIN and SIDEBOTHAM (2004): Fig. 9.1
Birrens, England	9x15	ROBERTSON (1975): 33, 58, 100
Castiellu de Llagú	9x5	BERROCAL et al (2002): 128–129
Richborough	9x7	BUSHE-FOX (1928): Fig. 1.1
Petavonium	9x7	CARRETERO VAQUERO (1998): Fig. 2.2
Abu Sha'ar	9x9/9x10	MULVIN and SIDEBOTHAM (2004): Fig. 9.1
Viladonga	11x12	FERNÁNDEZ PINTOS (2017): 236–237
Chao de Samartín	12x17	VILLA VALDÉS (2000): Fig. 8.1
Inverarvon	13x16	DUNWELL and RALSTON (1995): 562, 564

Fig. 3. Fragmentary quadrangular boards found within the borders of the Roman Empire.

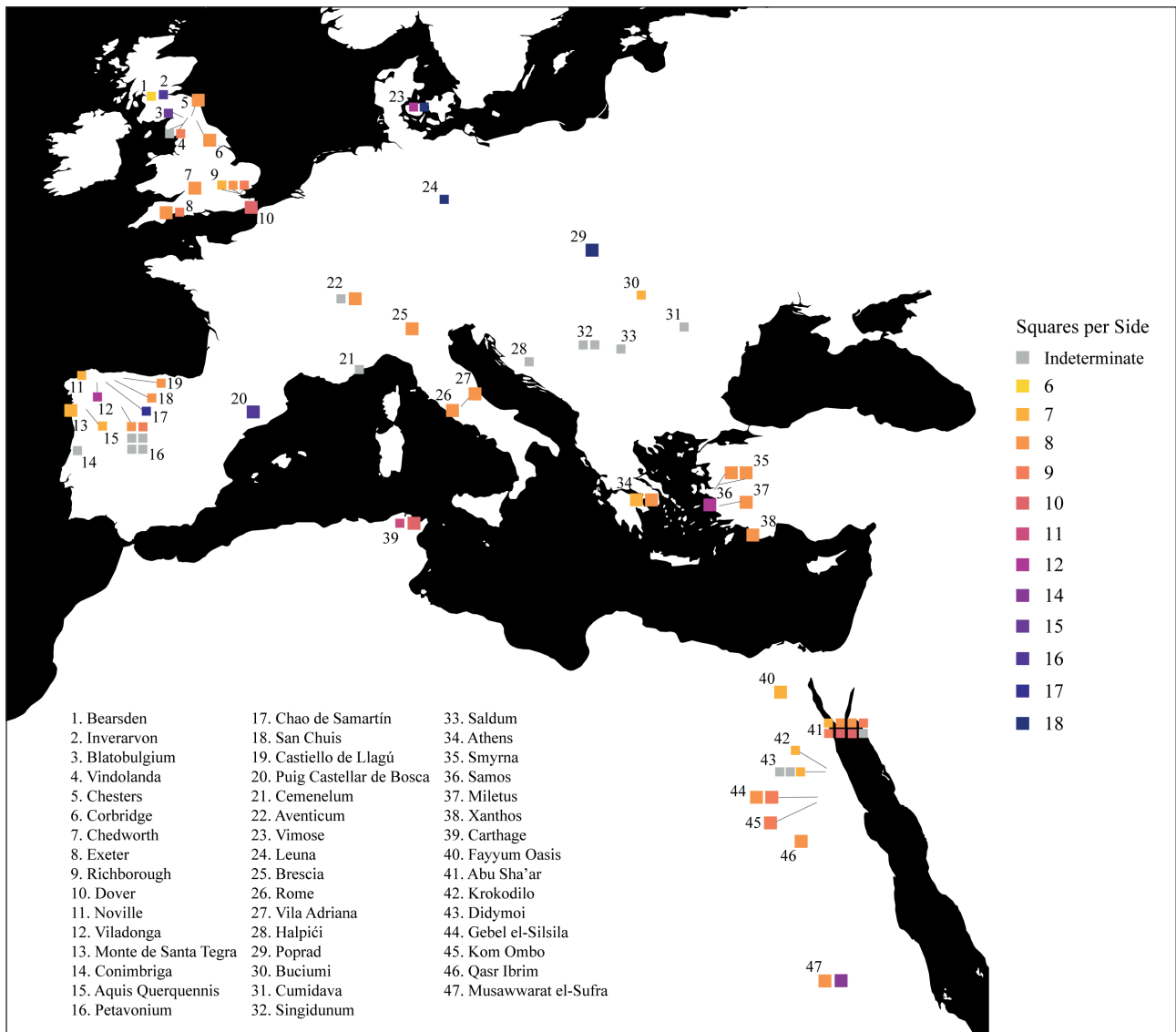


Fig. 4. Map of quadrangular boards contemporary with the Roman Empire. Large squares are complete boards; small squares are fragmentary boards.

sixteen. Nine of these boards were eight-by-eight, the most frequent size in the sample, with multiple examples of six-by-seven, seven-by-eight, and eight-by-nine boards. The boards with grids larger than this are unique examples of their respective patterns.

Information about the size of boards may also be inferred from the remains of fragmentary boards, in which the grid of squares is incompletely preserved and the exact pattern cannot be known (Figure 3). Many of these are too small to make any conclusions about their original size, but others clearly demonstrate that large boards existed in some places, with up to seventeen squares in a row.

Looking at the geographic distribution of the boards, it becomes apparent that the common board sizes are found throughout the empire, while the larger, uncommon board sizes are further away from the core of the Empire, defined as Italy and major urban centers (Figure 4).

2.3 Contemporaneous Quadrangular Boards

The archaeological record also presents examples

of boards which are contemporary with, but found outside of, the Roman Empire. These boards have been found at sites in Central and Northern Europe, well beyond the *limes* of the Empire, but certainly from areas which were in contact with the Romans. These boards all appear to have been quadrangular, like those seen in the Roman Empire. Indeed, the remains of one of these boards, found at Vimose in Denmark, bears the distinct pattern of the Roman game *Duodecim Scripta* on the opposite side of the grid,³² demonstrating that Roman games existed outside the Empire (see discussion on globalisation in Chapter 1). The preferred material for these boards seems to have been wood, and few of them are preserved though there are indications that, at least sometimes, these boards were placed in graves with counters on them, which may provide hints as to the original configuration, such as the one found at Leuna in Germany.³³ These boards are contemporary

32 KRÜGER (1982), 162, 222.

33 SCHULZ (1953), 29, 63–66, Pl. XXVII–XXVIII.

with the Roman Empire.³⁴ Only one example is intact: a wooden board with a seventeen-by-seventeen or seventeen-by-eighteen grid found in a 4th century CE elite tomb in Poprad, Slovakia.³⁵ A seven-by-fourteen board found at Musawwarat el-Sufra, a Meriotic site in Sudan,³⁶ also points to quadrangular boards of larger size outside the Empire.

The influence of Rome on the board games of temperate Europe is clear, especially with respect to game pieces, which were often imported from places within the Empire.³⁷ Indeed, the evidence for these games and the Roman influence that can be traced has been used as evidence for a connection between *Ludus Latrunculorum* and the medieval game *Hnefatafl* and other games which are probably related to it, such as *Brandubh* and *Tawlbwrdd*.³⁸ Aside from the material evidence for Roman games and Roman-style gaming materials in Northern Europe, the connection between *Ludus Latrunculorum* and *Hnefatafl* is made because it is thought that *Hnefatafl* also employed the custodial capture mechanism. This is inferred from the documentation of the game of *Tablut* among the Sámi people in what is now Finland during the 18th century CE by Linnaeus, which features custodial captures.³⁹ *Tablut* is thought to be a game derived from, or somehow related to, *Hnefatafl*, based on the board and the presence of a “King” piece for one player and not the other.⁴⁰ However, none of the sources which discuss *Hnefatafl* or its contemporary medieval games confirm that custodial capture was part of these games.

The archaeological evidence of quadrangular game boards found within the Roman Empire and in adjacent regions clearly demonstrate that boards ranging from six-by-seven to seventeen-by-seventeen or -eighteen were used to play games. This very wide range of board sizes presents the question of whether the same game can be played on such disparate boards, or if the different board sizes could indicate the presence of games other than *Ludus Latrunculorum* that were also played on quadrangular boards.

3. METHODOLOGY

3.1 Identifying Candidate Rulesets

In order to determine whether a game can be played on the wide range of quadrangular boards found in and around the Roman Empire, a reasonable set of rules must be applied to these boards to be able to

implement them in the *Ludii* software to calculate metrics that can indicate how the difference in board sizes changes the experience of the game. Candidate rulesets should contain all of the rules that are known to have been a part of *Ludus Latrunculorum* as described by the ancient sources, and have been documented geographically close to the region where the game was played. Ideally, contemporary rules would be chosen over rules which are further distant with regard to time, but since there are no completely documented game rules contemporary with the Roman Empire in the *Ludii* database, this is impossible.

To identify which games in the *Ludii* database contain the rules known for *Ludus Latrunculorum*, a search was made for the following game concepts: 3.3.2.6 Custodial Capture; 3.3.4.1.3 Orthogonal Direction (movement); 2.1.1.1.1 Square Shape (board); 2.1.1.2.1 Square Tiling (square spaces on the board); and 3.2.2 Pieces Placed Outside Board (indicates pieces begin off the board and must be placed on it, i.e. there is a placement phase in the game).⁴¹ This search produced five games that contain all of those rules: two types of *Gala*,⁴² *Kharebga*,⁴³ *Seega*,⁴⁴ and *Shantarad*.⁴⁵ Of these games, the only ones which were played in places that at one time were part of the Roman Empire are *Kharebga*, documented in El Oued, Algeria, and *Seega*, played in Egypt and Sudan. *Gala* is played on the island of Sulawesi in Indonesia, and *Shantarad* is played in Somalia, and thus far beyond the borders of the Roman Empire. Nevertheless, *Seega* was documented in the 19th century, and *Kharebga* in the 20th century, so the chronological distance between these games and that of *Ludus Latrunculorum* is greater than a millennium. Despite this, *Kharebga* and *Seega* are the most likely candidates for games that could be similar to *Ludus Latrunculorum*.

In addition, *Tablut* is the only game from Europe for which the complete set of rules have been documented that also contains the custodial capture mechanism, and there is reason to examine it on these boards as well. If *Tablut* is indeed related to *Hnefatafl*, as the evidence seems to suggest, this connects the evidence to a period in time much closer to the end of the Roman Empire, as evidence for *Hnefatafl* appears in the late 8th or 9th century CE. Nevertheless, *Tablut* does not have the placement phase known to exist in *Ludus Latrunculorum*, but has custodial capture, orthogonal movement, and a quadrangular board. However, it also has differentiated pieces, with one player playing with a “King” piece. There are also different numbers of pieces per player, a central

³⁴ DE VOOGT (2019), 92–93.

³⁵ STANEKOVÁ (2020), 56–57.

³⁶ CRIST *et al.* (2016a): 140.

³⁷ HALL, FORSYTH (2011), 1326–1330.

³⁸ HALL (2019), 204–207; HALL, FORSYTH (2011), 1333.

³⁹ LINNAEUS (1732), 147–148.

⁴⁰ MURRAY (1913), 445.

⁴¹ PIETTE *et al.* (2021).

⁴² MATTHES (1859), 899; MATTHES (1874), 71–72.

⁴³ BELLIN (1964), 53–54.

⁴⁴ DAVIES (1925), 138–139; LANE (1836), 356–357.

⁴⁵ MARIN (1931), 595–596.

square which affects capturing mechanisms, and different winning conditions for the two players—one attempts to move the King piece to the edge of the board, while the opponent attempts to capture the King piece. Despite these differences, and in light of the suggestions that Hnefatafl—and, therefore, Tablut—is derived from, inspired by, or in some way connected to Ludus Latrunculorum, it is worthwhile evaluating the Tablut rules on quadrangular boards as well.

3.2 Applying Rulesets to Boards

To measure behaviour metrics on the different board sizes of quadrangular board, and thus evaluate whether they all might have been used for the same set of rules, the rulesets for Kharebga, Seega, and Tablut were applied to boards with the dimensions of the intact quadrangular boards from within the Roman Empire, as well as the board from Poprad. These rulesets were implemented in the Ludii software so they could be played.

Kharebga is played on either a five-by-five board or a seven-by-seven board. Players alternate turns placing two pieces on the board, leaving the central space empty. When all the pieces have been placed, the players alternate turns moving their pieces over any distance in an orthogonal direction—also known as the slide move. Captures are made when an opponent's piece is between two of the player's own pieces (i.e., custodial capture). Captures are not made during the placement phase. The first player to capture all of their opponent's pieces wins.

Seega is very similar to Kharebga; it is played on a five-by-five, seven-by-seven, or nine-by-nine board. The other main difference is that pieces move one space at a time, rather than at a distance—known as the step move.

Tablut is very different from the other two. There is no placement phase; the pieces start in a fixed position, with the “King” piece beginning on the center spot, with its eight allied pieces surrounding it, and the opponent's sixteen pieces are arrayed toward the edge of the board. Players alternate turns moving a piece orthogonally any distance. Captures are custodial, but if the “King” is still in the central space, it can only be captured by surrounding it with four pieces, and if it is next to the central space, it must be surrounded on three sides. The central space cannot be entered by any piece including the “King” once it has left it. The player with the “King” wins by moving the “King” to the edge of the board; the opponent wins by capturing the King. It was played on a nine-by-nine board.

Adapting the rules to the known quadrangular boards contemporary with the Roman Empire immediately presents some issues. Only one of the known board sizes—seventeen-by-seventeen—provides a central space. Since the central space features in the rules

for all three games, the rules must be adapted for a board with an even number of spaces. For Kharebga and Seega, the rule that the central space must be left empty is discarded, and play begins when only two spaces are left empty on the board; with each player placing one piece on the final turn of the placement phase. It should also be noted that increasing the board size for these games implicitly introduces more pieces to the board: $(n/2)-1$ for each player, with n being the total number of spaces on the board.

For Tablut, the central space is more crucial, as it is required for the “King” piece's starting position which, in turn, imposes symmetry on the starting position of the pieces. The central space also imposes capturing and movement restrictions. Because of the many changes that would be required to adapt Tablut rules to a board with an even number of spaces, it was not applied to these boards, as it would effectively be a different game. Tablut rules were therefore only applied to the seventeen-by-seventeen board, and it was tested with two different versions: one with the original number of pieces—eight plus the “King” for one player; sixteen for the opponent— and one with the number of pieces increased in proportion and configuration to the larger board size—sixteen plus the “King” for one player, forty for the other.

3.3 Artificial Intelligence Agents

Once these rules were implemented on the different boards in Ludii, it was necessary to choose the parameters for the AI to conduct experiments to generate behavioural game-play metrics. Two basic tree search approaches were considered to provide the AI engines for automated game-playing; UCT⁴⁶ and Alpha-Beta⁴⁷ search. Both of these tree search approaches perform lookahead searches, in which they “look ahead” into various different future game states that may be reached through different

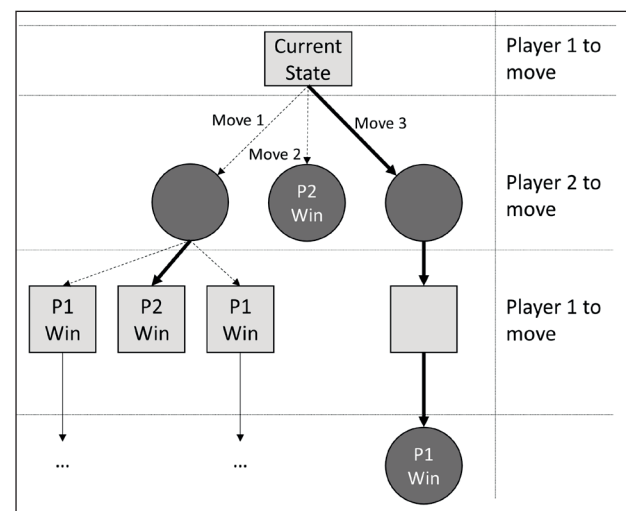


Fig. 5. Example game tree for lookahead tree search algorithms. Thick arrows correspond to moves that tree search algorithms would end up playing after sufficient analysis.

⁴⁶ KOCIS, SZEPESVÁRI (2006).

⁴⁷ KNUTH, MOORE (1975).

sequences of moves from the current game state (Figure 5), but they use different strategies for deciding which parts of their search spaces to explore or prioritise. UCT was found to barely play better than a random player, if at all, in all but the smallest of boards. This is a common issue for UCT in games where long sequences of random play—of which the algorithm uses many to estimate how valuable different game states are—are unlikely to lead to a variety of outcomes. This is the case in particular for the Seega and Kharebga rulesets, where random play on large boards is highly unlikely to ever lead to a victory for either player.

Alpha-Beta search was evaluated with a variety of different heuristic state evaluation functions, which it can use to compare states to each other without relying on such rollouts of random play. A straightforward “Material” heuristic, which incentivises the program to attempt capturing more opposing pieces than it loses, was found to produce an effective player. The program also used iterative deepening⁴⁸ to automatically tune its search depth for any given time constraint, and a transposition table⁴⁹ to improve the efficiency of the algorithm. The Alpha-Beta program can perform deeper tree searches if it is given more “thinking time” per move, and deeper searches usually lead to a stronger level of play. Hence, the amount of thinking time per move may be tuned to run different experiments between different players with different levels of playing strength.

One risk with AI-based players is that they will never get “bored”; if they fail to find a line of play that improves their position or leads to a win, but do find a line of play that simply stalls the game, they are likely to prefer indefinitely stalling the game over a more risky, aggressive move. This is arguably not representative of how humans would have played a game. In an attempt to address this, experiments were also run with variants of the Alpha-Beta program that were either restricted to solely even search depths, or solely odd search depths. When a program with a material heuristic searches only to odd search depths, it may typically be expected to play more aggressively, because it will focus on evaluating states in which it was allowed to make the final move—without considering a final reply by the opponent. Conversely, a program that searches only to even search depths may be expected to have a more defensive playstyle. This allows for experiments to be run in which it is expected that there will be at least some variety in playstyles between the two players, where the presence of at least one player with a more aggressive playstyle can help to reduce the likelihood of games that are stalled indefinitely.

⁴⁸ Korf (1985).

⁴⁹ Greenblatt, Eastlake, Crocker (1967).

3.4 Behaviour Metrics

By allowing these AI programs to play multiple games against each other, it is possible to measure certain gameplay properties that may give an indication about how the game is experienced by humans. These are called behaviour metrics, and can reveal interesting properties about a game which may not be apparent from just a casual observance of the rules—for example, measuring how long a game typically takes to play, the percentage of games which end in a draw, how many moves a player has to decide between each turn, etc. While there are over one hundred behaviour metrics currently implemented in Ludii, this paper is only concerned with three of them which produce interesting results:

- Duration: The average number of turns needed to complete a game. This metric can be used to tell if a game finishes in a reasonable number of turns, or will likely take thousands of turns to complete.
- Completion: The percentage of games which did not finish before reaching the 2,500 turn limit. Similar to duration, this metric can be used to tell if a game finishes in a reasonable number of turns.
- Branching Factor: The average number of different possible moves that a player can make during their turn. Games with a higher branching factor mean that each player has more options to consider on their turn, often indicating a more complex game.

3.5 Experiments

To compute all the metrics of each proposed ruleset, one hundred playouts are run between two Alpha-Beta agents, one using even search depths and the other using odd search depth alternately playing player one and player two after each game. Ten seconds are allocated to each agent to make a decision at each move. Each playout is limited by 1,250 moves per player; if that limit is exceeded the game is considered to be incomplete.

Every process was run on a single CPU core @2.2 GHz. 20,480MB of memory was allocated per process, of which 16,384 MB was made available to the Java Virtual Machine (JVM). Each process was also limited to four days of computation.

Concerning the largest board sizes (seventeen-by-seventeen, seventeen-by-eighteen for Seega and Kharebga rulesets), most of the playouts are extremely lengthy to run due to the high number of pieces and playable sites (~ 7.4 hours per playout). For this reason and due to the limit of four days, the decision was taken to run only ten playouts for these rulesets.

As a separate process to track the evolution of the number of pieces owned by each player after each move, one single playout for each ruleset was run and the total number of pieces placed on the board and

the number of pieces owned by each player placed on the board at each state were stored.

4. RESULTS

4.1 Completion

The Kharebga, Seega, and both Tablut rulesets demonstrate different trends with respect to board size. As board size increases, the percentage of playouts which played to completion before reaching the turn limit decreased for both Kharebga and Seega rulesets (Figure 6). Completion for Seega rulesets plummeted with increasing board size, with only the six-by-six, six-by-seven, and six-by-eight boards completing over 75% of the time. Boards larger than eleven-by-twelve completed less than 13% of the time. The seventeen-by-seventeen and seventeen-by-eighteen Seega rulesets never played to completion. In comparison, for the Kharebga rulesets, only the seventeen-by-eighteen board completed less than 75% of the time, with a gradual decrease with increasing board size. Both Tablut rulesets—only implemented on the seventeen-by-seventeen board—completed 100% of the time.

4.2 Duration

The number of turns for Kharebga, Seega, and Tablut rulesets showed results that mirror those for the completion metric (Figure 7). The length of the game increased steeply for Seega rulesets, with boards

greater than eleven-by-twelve in size at nearly 2,500 turns—the timeout limit. Kharebga rulesets also increased in duration with respect to board size, but the increase was more gradual. It should be noted that the rate of increase appears to be more drastic up to the eleven-by-sixteen board size, with the seventeen-by-seventeen and seventeen-by-eighteen boards having lower duration than expected in comparison to the trend on boards smaller than this. This could be attributed to sampling error with a sample of only ten playouts. The Tablut rulesets were shorter than any of the others, even though they were played on the second-largest board, lasting 26.04 turns for the unmodified Tablut rules and 73.17 turns for the ruleset with added pieces.

These results show that increasing board size has a greater effect on the duration of games played with Seega rules than for Kharebga rulesets. Nevertheless, though the increase in duration of Kharebga may be more gradual, increased board size does increase the duration of games to over one thousand turns. Meanwhile, Tablut rulesets, even though they are played on a large board, are played in a fraction of the amount of time as the Seega or Kharebga rulesets.

4.3 Branching Factor

Branching factor also increases for the Kharebga and Seega rulesets (Figure 8). The Kharebga rulesets all have greater branching factor than Seega rulesets

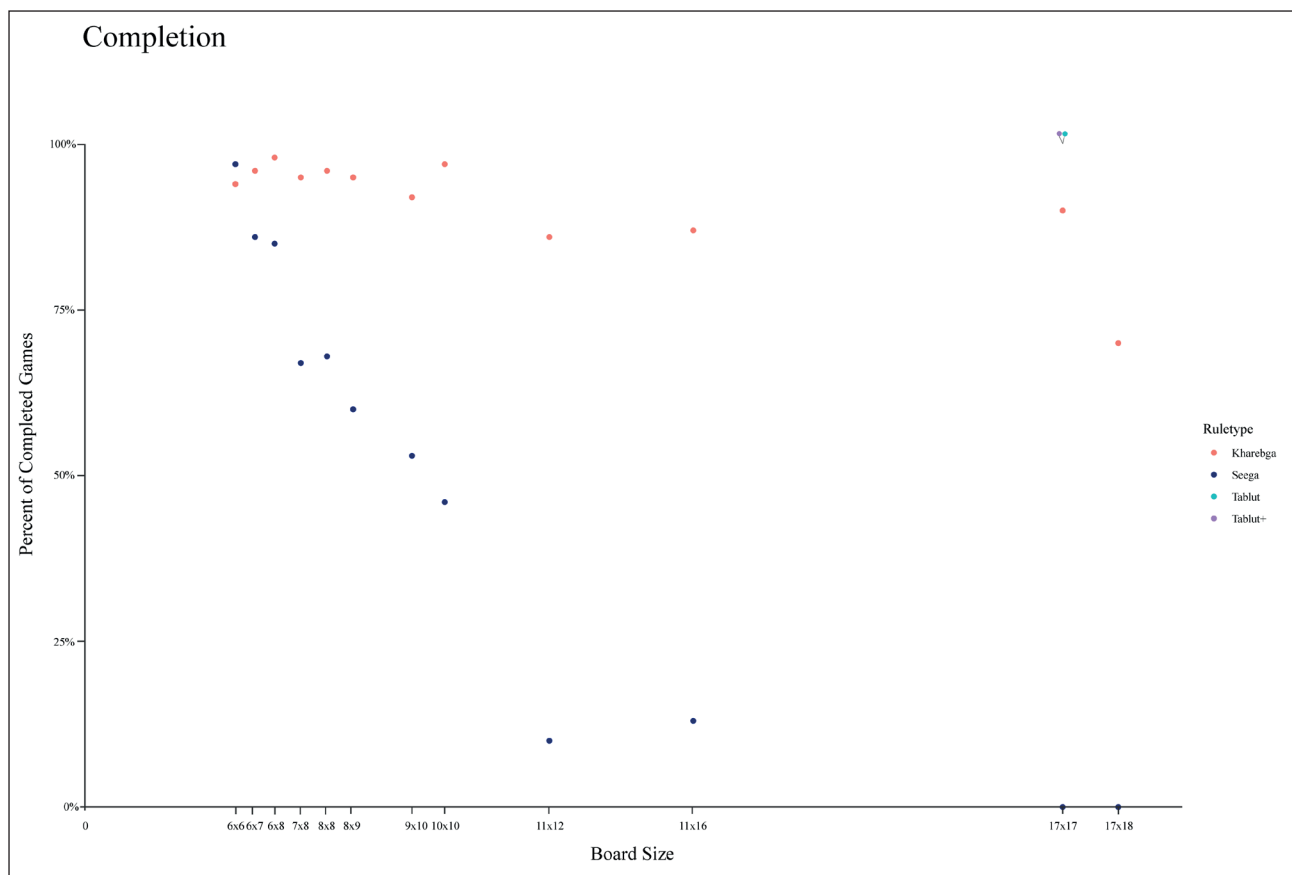


Fig. 6. Percent completion of playouts for different board sizes.

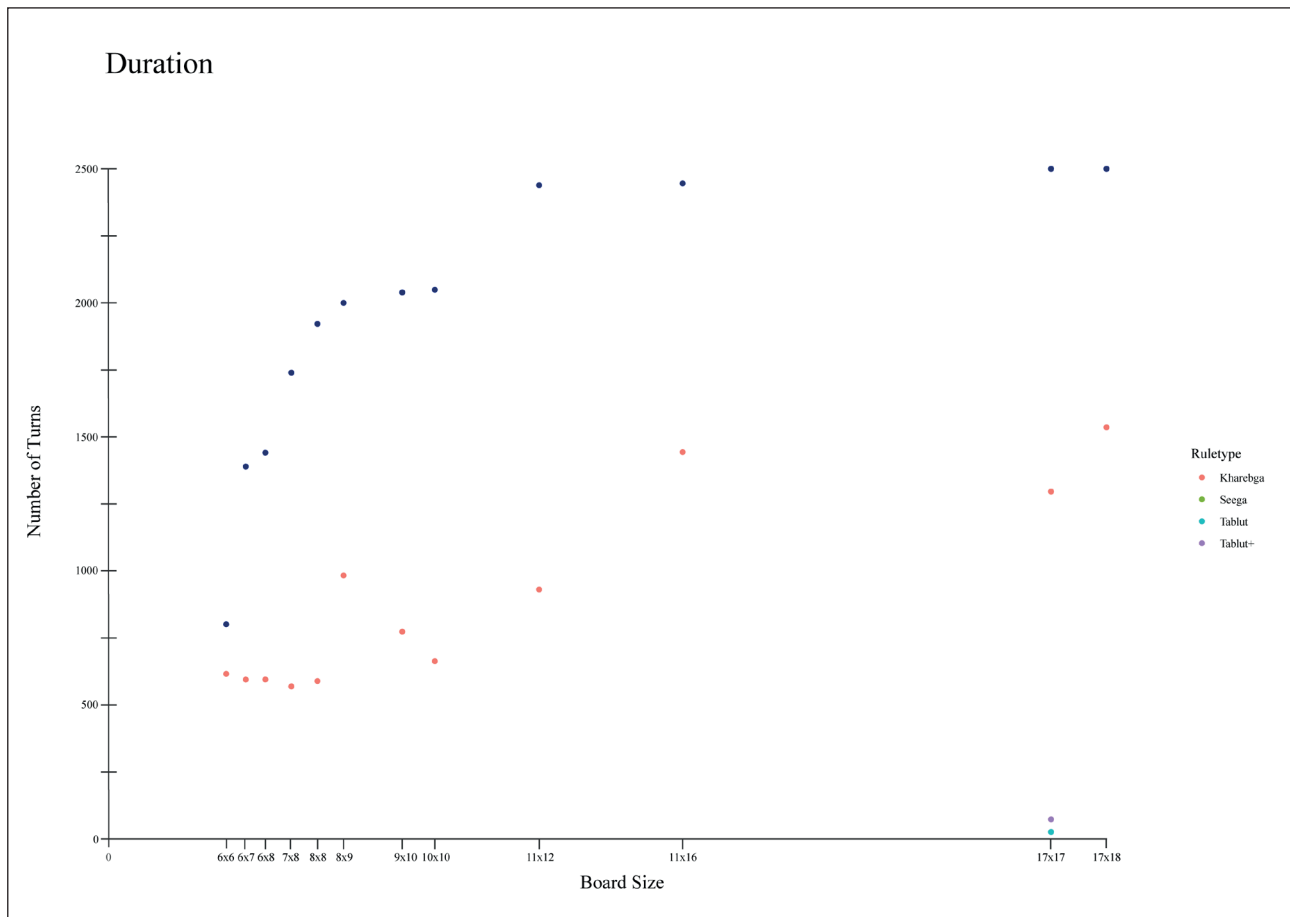


Fig. 7. Duration of playouts for different board sizes.

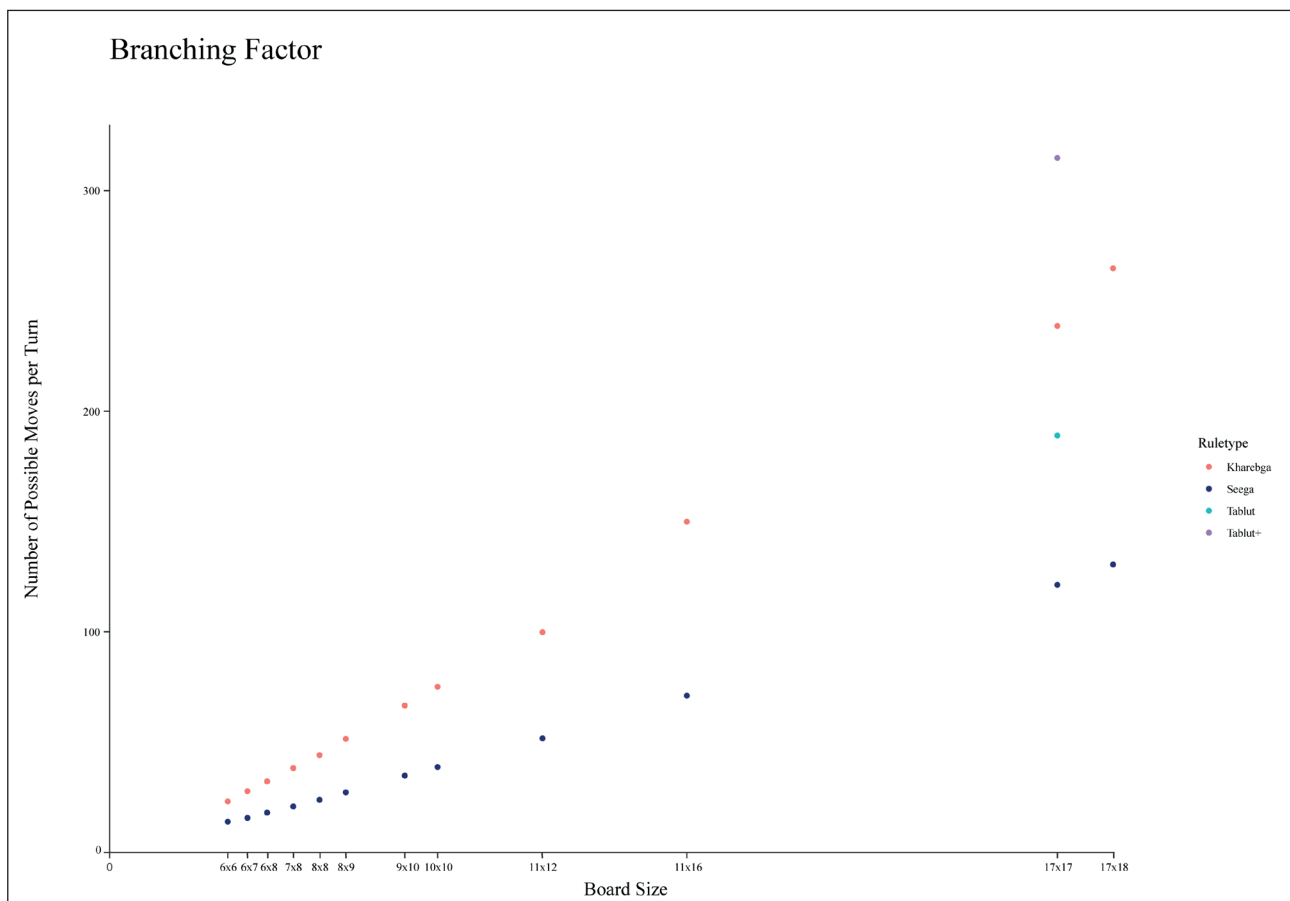


Fig. 8. Branching factor for different board sizes.

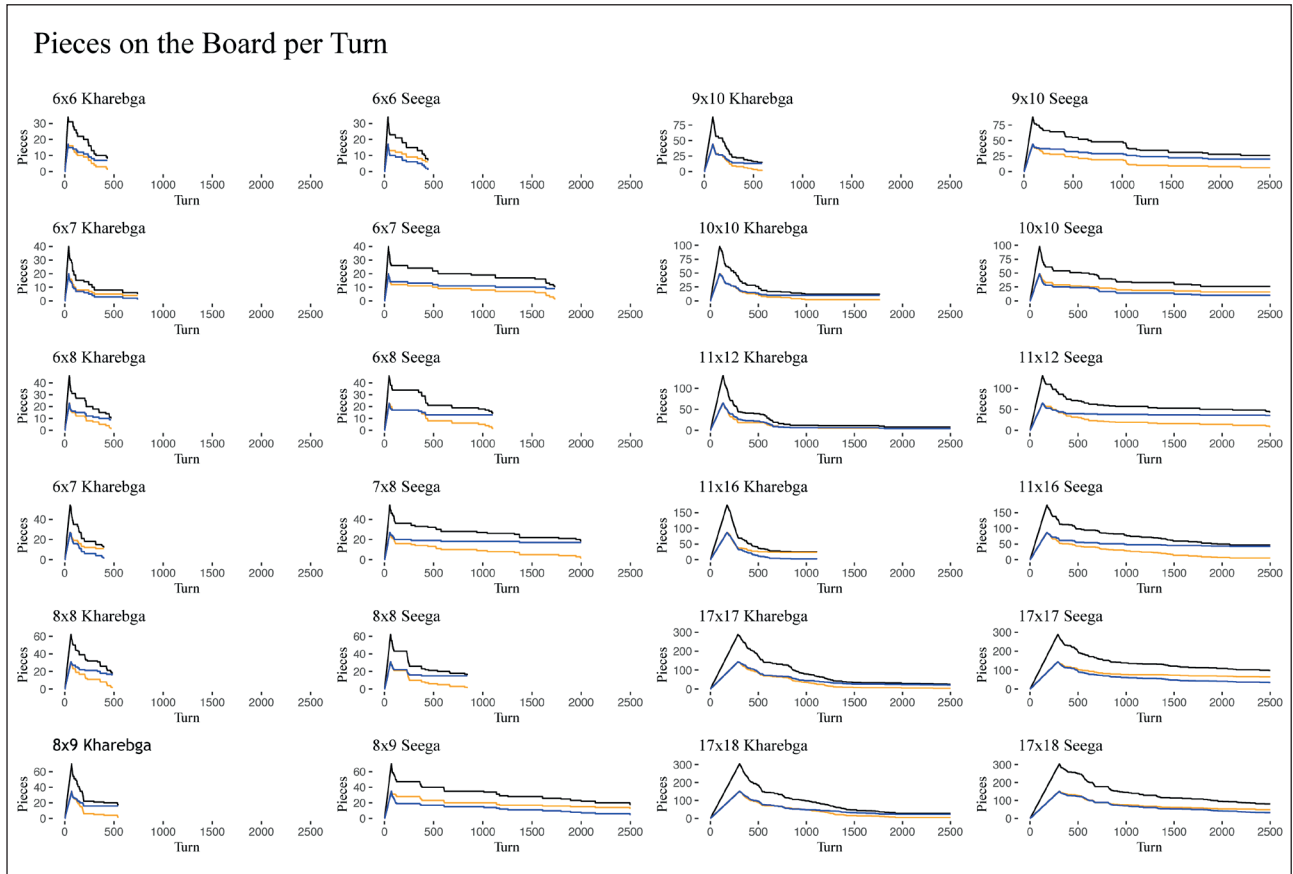


Fig. 9. Number of pieces per player at each turn for different board sizes.

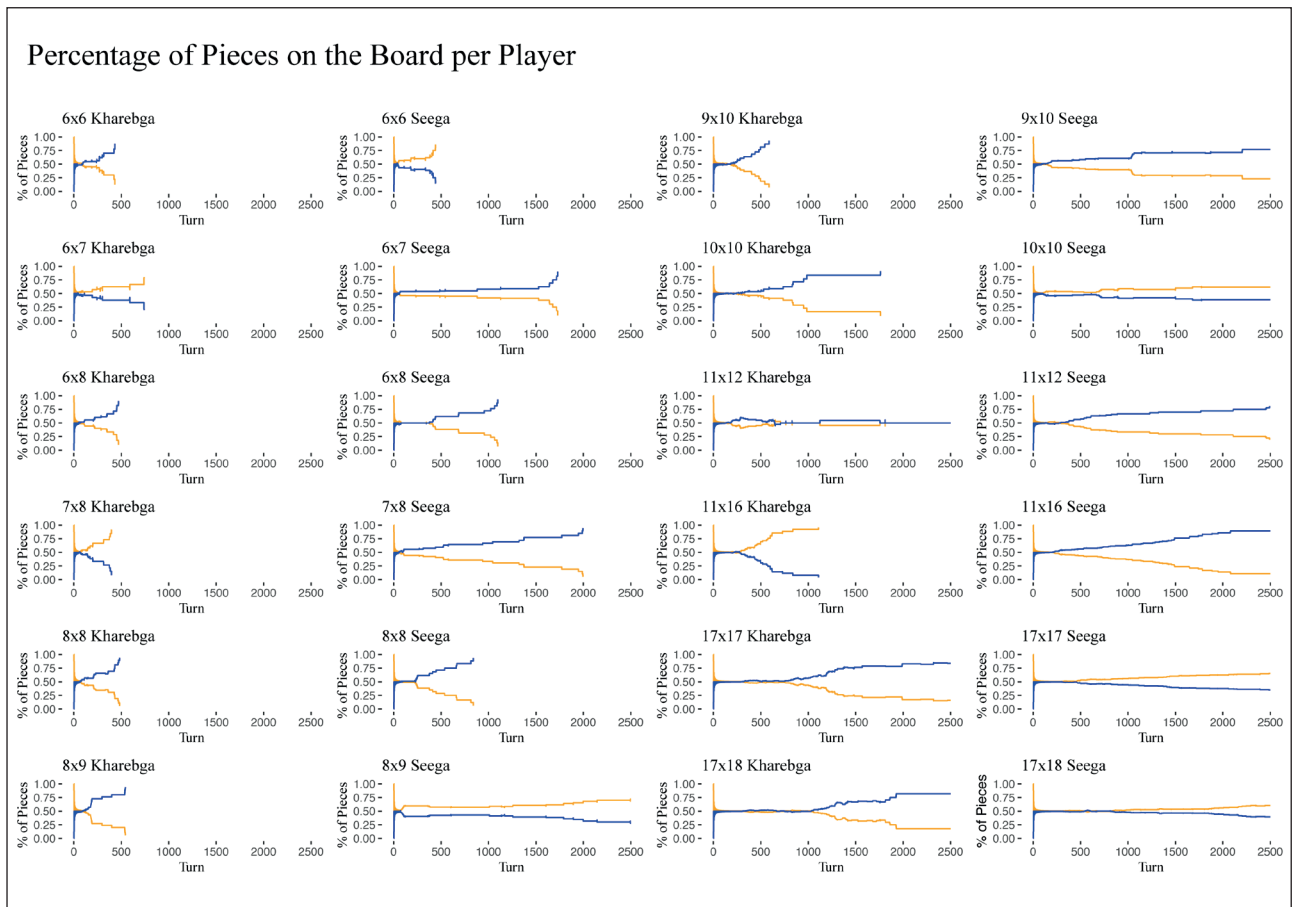


Fig. 10. Percent of pieces on the board for each player at each turn for different board sizes.

on the same board, and the difference increases with increasing board size. This is logical, since the sliding move of Kharebga gives more movement options for the players than the step move of Seega. Furthermore, the values for the Tablut rulesets show that these rules also have large branching factors, though they have fewer pieces than the Kharebga or Seega rulesets, but increasing the number of pieces (i.e., the difference between the Tablut and Tablut+ rulesets) also increases the branching factor.

4.4 Pieces per Turn

Examining the number of pieces on the board at every turn gives a sense of the gameplay for the Kharebga and Seega rulesets, and how that changes with increasing board size (Figure 9). The plots corroborate the duration metric results—that Kharebga rulesets play more quickly than the Seega rulesets. For all of the board sizes, and for both rulesets, the game tends to follow a similar pattern. After the initial placement phase, when the number of pieces on the board increases linearly, there is a period where captures can be made in fairly rapid succession. After this phase, there is a long tail where one player has more pieces than the other, but captures happen with decreased frequency, sometimes with hundreds of turns between captures. This tail lengthens more quickly with increasing board size in Seega rulesets, mirroring the more rapid increase in duration seen in this ruleset.

Examining the percentage of pieces owned by each player at each turn throughout the game provides more detail about gameplay (Figure 10). These results show more clearly that after the initial phase of rapid capturing, one player gains an insurmountable lead over their opponent. In the larger boards, hundreds of turns can be played where one player has twice or thrice the number of pieces of their opponent. In the one case where one player does not gain an advantage over the other (eleven-by-sixteen Kharebga), the game still continues for hundreds of turns without captures.

5. DISCUSSION

5.1 Playing on Large Boards

The playout data indicate some of the gameplay consequences of increasing board size for particular sets of rules. Based on what we know about Ludus Latrunculorum, the rulesets for Kharebga and Seega contain all of the rules that are known about the Roman game, and also are played in places where the Roman Empire once existed. Evaluating the playout results of these rulesets on the different board sizes found within the Roman Empire, as well as the board found at Poprad, can help us to recognize which rulesets may be closer to the original rules of Ludus Latrunculorum, and, when coupled with the

archaeological evidence, may indicate whether all of these boards were likely to have been used for Ludus Latrunculorum, a similar game, or a completely different one.

The clear results of the playouts were that both games increased in duration as board size was increased. This is not surprising, because increasing the size of the board also increases the number of pieces on the board, all of which need to be captured to win. However, the pieces per turn data show that increasing the board size also makes it more difficult for the AI agents to make captures, particularly in Seega rulesets. This likely happens for a couple of reasons. For Seega rulesets, which employ the step move, the AI may have difficulty detecting a move that brings it closer to an opposing piece in order to make a capture if they are distant from one another on the board because of the time limit imposed on the tree search. For Kharebga rulesets, the slide move allows the player more movement options, and therefore it is easier for an AI agent to avoid capture for a long time.

For these reasons, it is difficult to exactly translate the number of turns played by the AI agents into real-life turns of a game played by humans. Nevertheless, the trend of increased number of turns and board size is consistent across both rulesets, and constraints which compel the agents to play in a more human-like fashion do not eliminate the problem. Despite this, the problem is not only with the way AI agents play, but is a problem with having a large amount of empty space with few pieces remaining on the board, requiring the coordinated movement of two pieces to make a capture. This is alluded to in previous work refuting the hypothesis that the so-called “Doctor’s Game” from Stanway, England was a Ludus Latrunculorum board because there were too few pieces included with the board.⁵⁰ Furthermore, the difficulty in making the final captures has been observed in classroom settings,⁵¹ indicating that this is a real-world problem with the game and not merely a limitation of the AI agents.

The fact that the Tablut rulesets play much more quickly on a large board than the Kharebga and Seega rulesets shows that a large board itself is not the problem, nor is it the number of choices that the player has to make, since the branching factor for the Tablut and Tablut+ rulesets also have high values for this metric. It may be, though, that having a large number of decisions to make, over hundreds of turns without captures, would be exhausting and indicates a game that would likely not be played. For these reasons, Seega is a less convincing candidate ruleset for the larger boards. Since we know that

⁵⁰ SCHÄDLER (2007), 368–369.

⁵¹ Marco Tibaldini, personal communication.

Seega was played on nine-by-nine boards, it cannot be ruled out as a possibility for the smaller boards—but the fact that it was played on boards ranging from five-by-five to nine-by-nine but not larger may be indicative of the fact that larger boards make the game interminable with these rules.

Furthermore, the slide move employed in Kharebga is more in line with Ovid's description of the pieces moving in a straight line than a step move. It is also important to note a passage in the *Historia Augusta*, which provides a second-hand account of Proculus being declared *imperator* after winning ten games of Ludus Latrunculorum in a row,⁵² indicating the game must have been of sufficient brevity to play so many in succession. For all of these reasons, it appears to be more likely that Kharebga was a more suitable ruleset because the slide move allows for a game that can more frequently be played to a conclusion, and which can be played faster on the greatest number of boards. Neither Seega nor Kharebga appear to play in a reasonable fashion on larger boards. The Tablut and Tablut+ payout data indicate that other rules can be more amenable to large boards.

5.2 Games with Large Boards

It is useful to look at games that were played on large boards to examine whether people played games similar to Kharebga and Seega on large boards. The sample of games documented as part of the Digital Ludeme Project allows for such comparison. In the sample of 1,007 games at the time of writing, 17 war games⁵³ have boards with more than 100 playing sites. These can largely be divided into four separate categories: Draughts games, multiplayer games, enlarged versions of existing games with added pieces with new movement properties, and Konane. Each of these categories features something about their rules that either requires a large board or speeds up the game. Draughts games only use half of the sites. Multiplayer games have three or more players, requiring more space than a game with two players. Enlarged games, which are largely versions of Chess and Shogi, add pieces which both necessitate a larger board for the starting position and introduce more powerful movement for these pieces. Finally, the number of moves in Konane is capped by the number of spaces on the board because each move must involve a capture.

In addition, there is archaeological and historical evidence for other large games, for which the rules have been lost. Hnefatafl, and games which may be related to it such as Alea Evangelii, fall into this group. Hnefatafl could apparently be played on a thirteen-by-thirteen board, while Alea Evangelii was

depicted on an eighteen-by-eighteen board.⁵⁴ The payout results for the Tablut rules are relevant here, since it is likely that Hnefatafl and Alea Evangelii had similar rules. When applied to larger boards, Tablut rules play much faster than the Seega or Kharebga rules, which may be attributable to the fact that winning the game is focused on the capture or escape of the “King” piece. It is expected that Hnefatafl and Alea Evangelii would play similarly.

The game at Stanway may have been a nine-by-thirteen board. Though Roman in date, it is thought to have been a Celtic board game.⁵⁵ The aforementioned boards from Vimose include one which has eighteen squares per row and another with fourteen or more per row.

Other boards, for which the names of the games are unknown, have been found at pre-Roman sites in the Mediterranean basin. At Tell Zakariya in the Levant, a complete twelve-by-twelve board was found in the upper layers of the site, post-Iron Age but pre-Roman in date, as well as a fragment of a stone gaming table that had at least ten rows of squares.⁵⁶ Another, at nearby Maresha, has at least ten spaces per row, and is Hellenistic in date.⁵⁷ Other large pre-Roman boards have been found in Greece at Rhamnous, where a nine-by-nine and an eight-by-ten board were found inscribed on a block dating to the 3rd century BCE,⁵⁸ as well as an eleven-by-eleven Hellenistic board from Pella.⁵⁹

These boards show that other games were played on larger boards before and after the Roman Empire. The Hellenistic boards above could be for the game Polis, which is mentioned in Greek sources but not described in great detail. It is thought to be similar in some ways to Ludus Latrunculorum, but the candidate boards are larger than the commonly-found Ludus Latrunculorum boards and, perhaps indicatively, tend to have an odd number of spaces while Ludus Latrunculorum strictly has an even number. Less is known about the rules for Polis, so there is likely something missing from these rules to make a playable game on a larger board.

5.3 Archaeological Context of Boards

The archaeological context of the boards provides further evidence for the interpretation of quadrangular grids. Looking at the geographical distribution of board size, it is clear that the most common board sizes, six-by-seven, seven-by-eight,

52 Flavius Vopiscus Syracusanus, *Four Tyrants*, 12.1–3.

53 Defined as games in which capturing the opponent's pieces is the primary criterion for determining a win.

54 DUGGAN (2021); SCHULTE (2017) FOR RECENT SURVEYS OF THE EVIDENCE FOR THESE GAMES.

55 SCHÄDLER (2007), 368–369.

56 BLISS, MACALLISTER (1902), 144.

57 STERN (2019), 127–128; Stern this volume.

58 FACHARD (2021).

59 IGNATIADOU (2019), 142, 152.

and eight-by-eight, are widely spread throughout the empire. These are also found in major cities of the empire, such as Athens, Rome itself, and the cities of western Asia Minor. These games exist as graffiti in urban contexts, such as at the Basilica Iulia and the Parthenon, but others were also found in military contexts, particularly in Britain, Spain, and Egypt (e.g., Chesters, Monte de Santa Tegra, Abu Sha'ar). The widespread use of these particular grid patterns, and the fact that the seven-by-eight and eight-by-eight boards are the only ones found intact in Italy, follows the pattern that would be expected in the Roman Empire—that the game would appear both in the central core of the Empire as well as in the places the Romans occupied.

The fact that the larger boards are not found in Italy, but typically in forts on the fringes of the empire, could be explained in two ways. The simplest argument is that troops stationed at forts have a lot of idle time, and therefore the presence of larger boards at these places might explain an attempt to take up this time. The playout data indicates, though, that the rules known for Ludus Latrunculorum played on these boards are probably not amenable even for people who are trying to waste a considerable amount of time.

Perhaps a more compelling explanation is that these larger game boards are not for Ludus Latrunculorum, but other games about which we know nothing concerning the rules. Pre-Roman quadrangular boards indicate that Ludus Latrunculorum was not the first such game to exist in the Mediterranean basin. Indeed, Polis, which was played in the Greek world, seems to have been played on larger boards, and it is possible that the board from Samos is this game, rather than Ludus Latrunculorum. But other boards, such as one from the Late Period Sacred Animal Necropolis at Saqqara in Egypt,⁶⁰ as well as several in northwest Spain⁶¹ cannot be tied to any known games. Coupled with the conclusion that the Stanway game was a Celtic board-game of unknown type, and that other larger boards found at Vimose, Leuna, and Poprad were also probably indigenous rather than Roman games,⁶² it is plausible that these larger boards were not Ludus Latrunculorum, but other games which existed in a suite of games played on quadrangular boards stretching from Britain and Scandinavia to the Meroitic Kingdom in Sudan. Auxiliary troops were drawn from the indigenous populations of the Empire and beyond,⁶³ so the presence of indigenous games at Roman military sites is not surprising, particularly since games

function as social lubricants which can be used to acquaint people with one another and to form the basis for other social interactions.⁶⁴

6. CONCLUSIONS AND FUTURE WORK

Computational methods can add another line of evidence to traditional archaeological and philological methods to help identify board games and test plausible rulesets. For Ludus Latrunculorum, playout analysis shows that a game in which pieces move with a slide movement such as Kharebga fits the description of the Roman authors and is reasonably playable on the boards known from the archaeological record. Furthermore, larger boards seem to be less amenable to these rules, and traditional games with similar rules on large boards have not been recorded in human history. This leads to one of two conclusions: that there are key rules missing from the written record of Ludus Latrunculorum that would allow these rules to work on large boards, or that there are other games with unknown rules that were played on large boards. The archaeological record supports the presence of pre- and non-Roman games on large boards, so the latter argument is favoured.

Future work can improve the utility of computational methods in the study of games. Developing improved AI agents by introducing heuristics and features to the agents can help them to play more effectively. The development of game distance metrics can help to quantitatively analyze the ludemic similarity of games to one another, allowing for better identification of candidate rulesets. In addition, a social network approach can add the cultural dimension to measurement of game distance. At press, this is being developed by the Digital Ludeme Project to potentially automate the identification of candidate games for analyzing and reconstructing traditional games of the past. These innovations can be used not only to bring new insight into antiquity, but also to make the past interactive and to preserve this intangible cultural heritage by providing playable games that more closely replicate the way ancient peoples played board games.

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60 MARTIN (1981), 30, Pl. 39.

61 BERROCAL-RANGER *et al.* (2003), 103–104; LLANOS ORTIZ DE LANDALUZE (2002), 191–193.

62 SCHÄDLER (2007), 369–373.

63 HAYNES (2013), 95–142.

64 MALABY (2003), 59–74; CRIST *et al.* (2016b).

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